



# Parallelizing Assignment Problem with DNA Strands

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**Background:** Many problems of combinatorial optimization, which are solvable only in exponential time, are known to be Non-Deterministic Polynomial hard (NP-hard). With the advent of parallel machines, new opportunities have been emerged to develop the effective solutions for NP-hard problems. However, solving these problems in polynomial time needs massive parallel machines and is not applicable up to now.

**Objectives:** DNA (Deoxyribonucleic acid) computing provides a fantastic method to solve NP-hard problems in polynomial time. Accordingly, one of the famous NP-hard problems is assignment problem, which is designed to find the best assignment of  $n$  jobs to  $n$  persons in a way that it could maximize the profit or minimize the cost.

**Material and Methods:** Applying bio molecular operations of Adelman Lipton model, a novel parallel DNA algorithm have been proposed for solving the assignment problem.

**Results:** The proposed algorithm can solve the problem in time complexity, and just  $O(n^2)$  initial DNA strand in comparison with initial sequence, which is used by the other methods.

**Conclusions:** In this article, using DNA computing, we proposed a parallel DNA algorithm to solve the assignment problem in linear time.

**Keywords:** Adelman Lipton model, Assignment, DNA computing, DNA algorithm, Molecular computation.

## 1. Background

Deoxyribonucleic acid (DNA) is a molecule that carries most of the genetic instructions used in the development, functioning, and reproduction of all known living organisms and many viruses. Just one gram of DNA can hold about 680 peta bytes ( $6.8 * 10^{14}$ ) equal to one billion (1) Compact Disc (CD), which take 163,000 centuries to be listened.

Considering parallelism aspect, it also is capable of parallelizing molecules at a time(2), and this enormous parallelism persuades the researchers to build a model by the use of DNA strands to solve NP-hard problems (3). Considering the similarities in biological and mathematical operations, and the facts that DNA is stable and also predictable in its reactions, this macromolecule can be used to encode information for mathematical systems.

In 1994, Adelman (4) designed the first model of molecular computation by solving a seven-node instances of Hamiltonian path problem, and then showed the enormous parallel power of DNA computation. In 1995, Lipton (5) solved satisfiability problem by Adelman's method and designed a new method called Adelman-Lipton model, which was consisted of three levels as follows:

a) Producing specific sequences and putting them in a test tube.

b) Performing a number of operations on the DNA sequences inside the tube and removing unwanted sequences, which certainly cannot be considered as the final answer of the problem.

c) Reading and reporting the DNA sequences, if any exists, in the final tube as final answer.

Ouyang (6) solved Maximal clique problem in 1997 by designing *Restriction enzyme model*. Roweis designed *sticker model* (7) in 1998. The *Self-assembly model* was designed by Winfree in 1998 (8). The *Surface based model* (9) was designed by Smith in 1998. And finally, Sakamoto proposed the *Hairpin model* in 2000 (10).

As the other applications of DNA computing we can point to Zhang's image encryption algorithm (11) or Patel multiple image encryption (12) which is based on DNA computing or novel data encryption scheme which is proposed by Patro (13) by using DNA computing.

Many efforts have been done by researchers to solve NP-hard problems via these models such as knapsack (14), n-queen (15), binpacking (16), maximal clique problem (17), graph vertex coloring problem (18), maximal matching problem (19), maximal connected subgraph problem (20), maximum complete bipartite

subgraph (21), maximum independent set problem (22), maximum matching problem (23), maximum k-colourable subgraph (24), hamiltonian path (25, 26), minimum k-suppliers (27).

## 2. Objectives

Assignment is another famous NP-hard problem (28), which has been solved in different versions by researchers as Wang (29) who solved the unbalanced version of assignment, or Yang (30) who solved the quadratic version of assignment.

Based on Adleman-Lipton model, we proposed a parallel DNA algorithm with time complexity for assignment problem, and initial DNA strand in comparison with initial DNA strands of similar solutions such as that was proposed by Wang et al (31).

## 3. Material and Methods

In Assignment problem, we have n jobs, which can be performed by n people. Each of the people will do each one of the jobs with a specific wage. The objective of this study was to assign each job to one person in such a way that the total wages become minimized.

As an example, consider four teachers ( $T_1$ - $T_4$ ) from four different cities ( $C_{T1}$ - $C_{T4}$ ) and also four universities ( $U_1$ - $U_4$ ) in four different other cities ( $C_{U1}$ - $C_{U4}$ ).

Table 1 shows the ticket price for each flight from cities  $C_{T1}$ - $C_{T4}$  to cities  $C_{U1}$ - $C_{U4}$ . The goal is to send each teacher to one of the universities in a way that the total price of purchased ticket becomes minimized.

Table 1. Ticket prices for the example described in the text.

City	$C_{T1}$	$C_{T1}$	$C_{T1}$	$C_{T1}$
$C_{U1}$	40	140	90	180
$C_{U2}$	120	20	70	110
$C_{U3}$	80	25	25	115
$C_{U4}$	80	180	130	140

We can represent Table 1 as a cost matrix:

$$\begin{bmatrix} 40 & 140 & 90 & 180 \\ 120 & 20 & 70 & 110 \\ 80 & 25 & 25 & 115 \\ 80 & 180 & 130 & 140 \end{bmatrix}$$

One possible assignment is as follow:

$$\begin{bmatrix} 40 & 140 & \mathbf{90} & 180 \\ 120 & 20 & 70 & \mathbf{110} \\ 80 & \mathbf{25} & 25 & 115 \\ \mathbf{80} & 180 & 130 & 140 \end{bmatrix}$$

Where selected flights highlighted as yellow. The total cost of this assignment is: 90\$ + 110\$ + 25\$ + 80\$= 305\$. But the best possible assignment is:

$$\begin{bmatrix} \mathbf{40} & 140 & 90 & 180 \\ 120 & \mathbf{20} & 70 & 110 \\ 80 & 25 & \mathbf{25} & 115 \\ 80 & 180 & 130 & \mathbf{140} \end{bmatrix}$$

Which will cost 40\$ + 20\$ + 25\$ + 140\$= 225\$ and means to send  $T_1$  to  $U_1$ ,  $T_2$  to  $U_2$ ,  $T_3$  to  $U_3$ , and  $T_4$  to  $U_4$ .

### 3.1. Adelman-Lipton Model

Different operations of Adelman-Lipton model:

- Merge ( $T_1, T_2$ ): The content of tube  $T_1$  will be merged with  $T_2$  and will also be moved into  $T_1$ .
- Copy ( $T_1, T_2$ ): The content of  $T_1$  will be copied into the empty tube  $T_2$ .
- Detect (T): Determining whether any sequence exists in tube T or not.
- Extract ( $T_1, X, T_2$ ): The subset of  $T_1$ , which contains the X sequence will be transferred into  $T_2$ .
- Select ( $T_1, X, T_2$ ): The subset of  $T_1$  containing sequences longer than X will be transferred into  $T_2$ .
- Selecting ( $T_1, L, T_2$ ): It will select sequences of size L from tube  $T_1$  and put them in tube  $T_2$ .
- SelectMax ( $T_1, T_2$ ): The longest sequence in tube  $T_1$  will be chosen and moved into tube  $T_2$ .
- SelectMin ( $T_1, T_2$ ): The shortest sequence in tube  $T_1$  will be chosen and moved into tube  $T_2$ .
- Annealing (T): ssDNA will be converted to dsDNA by freezing the tube.
- Denaturing (T): dsDNA will be converted to ssDNA by freezing the tube.
- Discard (T): The content of tube T will be eliminated.
- PCR ( $T, T_1$ ): A lot of copies of DNA strands will be made and placed in tube  $T_1$ .
- Append Tail (T, Z): Sequence Z will be appended to the end of all sequences in tube T.
- Read (T): A sequence of tube T will be read.

### 3.2. DNA Algorithm for Assignment Problem

First of all, all  $n^n$  combinations of people and jobs will be generated as different DNA strands. Then, the strands having all people will be separated and the others will be discarded. After that, among remained stands, those that have all jobs will be separated and the others will be discarded. The shortest sequences will be regarded as the final solutions.

The proposed algorithm consists of six steps

- Building  $2n$  unique 10-mer sequences (sequence with length 10) for each of n people and n jobs.
- Building all  $n^2$  possible combinations of jobs with people



- For (i in 2 to n)
    - For (j in 1 to n)
      - PCR (T, T<sub>i</sub>)
      - AppendTail (T<sub>j</sub>, P<sub>i</sub> J<sub>j</sub>)
      - Merge (Temp, T<sub>j</sub>)
      - Discard (T<sub>j</sub>)
    - Discard (T)
    - Copy (Temp, T)
    - Discard (Temp)
- The trace of the top algorithm for three people and three job is shown as an example in **Table 4**.

**Table 4.** Trace for three people and three jobs

Round	Tube T <sub>1</sub>	Tube T <sub>2</sub>	Tube T <sub>3</sub>	Tube T
i=1	P <sub>1</sub> J <sub>1</sub>	P <sub>1</sub> J <sub>2</sub>	P <sub>1</sub> J <sub>3</sub>	P <sub>1</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub>
i=2,j=1	P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>1</sub>			P <sub>1</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub>
i=2,j=2		P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>2</sub>		P <sub>1</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub>
i=2,j=3			P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>3</sub>	P <sub>1</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub>
i=3,j=1	P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>1</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>1</sub> , P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>1</sub> , P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>1</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>1</sub>			P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>1</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>2</sub> , P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>2</sub> , P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>3</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>3</sub>
i=3,j=3			P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>3</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>3</sub> , P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>3</sub> , P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>3</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>3</sub>	P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>1</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>2</sub> , P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>2</sub> , P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>3</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>3</sub>
Final	P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>1</sub> , P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>1</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>2</sub> , P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>2</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>1</sub> P <sub>3</sub> J <sub>3</sub> , P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>2</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>1</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>2</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>3</sub> ,P <sub>1</sub> J <sub>3</sub> P <sub>2</sub> J <sub>3</sub> P <sub>3</sub> J <sub>3</sub>			

Eliminate all sequences, which do not consist of all people.

- For (i in 1 to n)
  - Extract (T, P<sub>i</sub>, Temp)
  - Discard(T)
  - Copy (Temp, T)
  - Discard (Temp)

Eliminate all sequences, which do not consist of all jobs.

- For (i in 1 to n)
  - Extract (T, Job<sub>i</sub>, Temp)
  - Discard(T)
  - Copy (Temp, T)
  - Discard (Temp)

Final Answer

- SelectMin (T, Temp)
- Read (Temp)

#### 4. Results

4.1. The solutions of assignment problem with n jobs and individuals can be obtained by above mentioned DNA molecules operations.

**Proof.** At first, all combinations of the n job assignments were generated in the sample space. Then, the sequences containing all people were kept and this guaranteed that none of the people are jobless. And finally, among the remaining sequences, the sequences containing all the jobs were kept and this guaranteed that all of the jobs will be done at the end. Therefore, for sure, all possible assignments are in final tube, and as the cost of each assignment was attached to each of them in the strain design phase, so the strand with minimum length

among the strands of final tube would be the minimum cost assigning solution.

4.2. *The time complexity of the proposed algorithm for solving assignment problem with n jobs and individuals is  $n^2$ .*

**Proof.** As the complexity of every biological operation is  $O(1)$  (33), the time complexity of algorithm can be calculated by adding the time complexity of all steps as follows:

T (Sample space):  $O(n)$  for the first loop and  $O(n^2)$  for the nested loop.

T (Persons): Four biological operations in each step, which become  $O(1)$ , and in n step it would be  $O(n)$ .

T (Jobs): Four biological operations in each step, which become  $O(1)$ , and in n step it would be  $O(n)$ .

T (Last step): Two biological operations, which become  $O(1)$ .

So, the time complexity of the algorithm will be arrived by equation 1 as follows:

*Equation 1 : Time Complexity*

$$T(n) = O(n) + O(n^2) + O(n) + O(n) + O(1) = O(n^2)$$

## 5. Discussion

In the proposed method, by using adenine, thymine, and cytosine; we produced distinct 10mer sequences equal to the number of people and jobs of assignment problem. Then, by using polymerase chain reaction on them, we produced  $n^2$  sequences equal to the dot product of n persons to n jobs. Finally, by adding guanine equal to the cost of assigning a job to a person to the related sequence, we got the desired sequence.

Wang *et al.*(31), Kang *et al.*(34), Shu *et al.*(35), and Ebrahim *et al.*(36), Tsafaris *et al.*(37), Rashid *et al.* (38) all used the same model to solve the assignment problem using  $n^n$  distinct sequence with the time complexity of  $O(n^2)$ . The proposed algorithm reduced the initial cost to a great extent by using just 2n initial sequences in comparison with  $n^n$  initial sequences, which is used by the other methods and solve the problem with the same time complexity.

## 6. Conclusion

Ultra-efficient parallelism of the methods of DNA computing in contrast with the obvious limitations in storage, speed, intelligence, and miniaturization of electronic computers is considerable. Although there are still some problems that need a further study in biologic technology, it is still possible to solve a lot of NP-hard problems in linear time via DNA computing. In this article, we highlighted a DNA computing model with biological operations based on Adelman-

Lipton model to solve Assignment problem with the time complexity of  $O(n^2)$  with just buying 2n 10mer sequences and making the other needed sequences in our lab in comparison with the previous methods which need a lot of different sequences.

We hope that, in near future, molecular computer become usable instead of electronic computers, which can cause DNA computing solutions become applicable.

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